Data-Driven Radial Compressor Design Space Mapping

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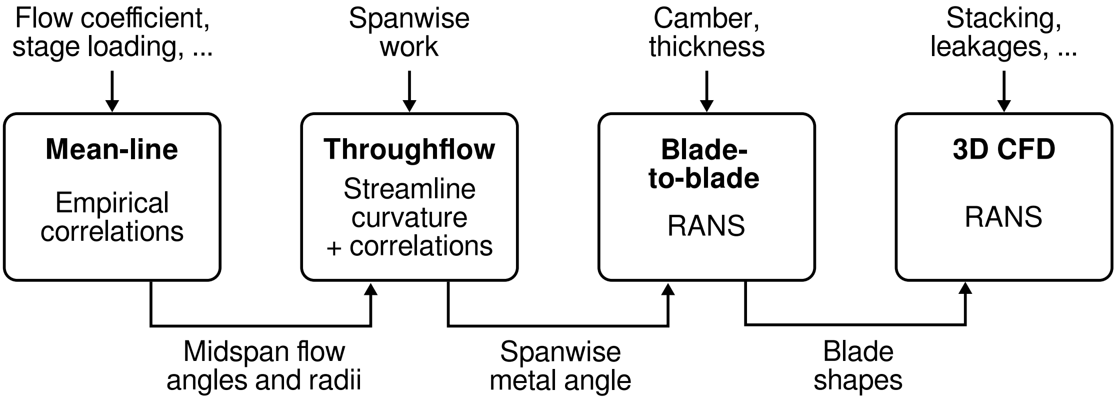
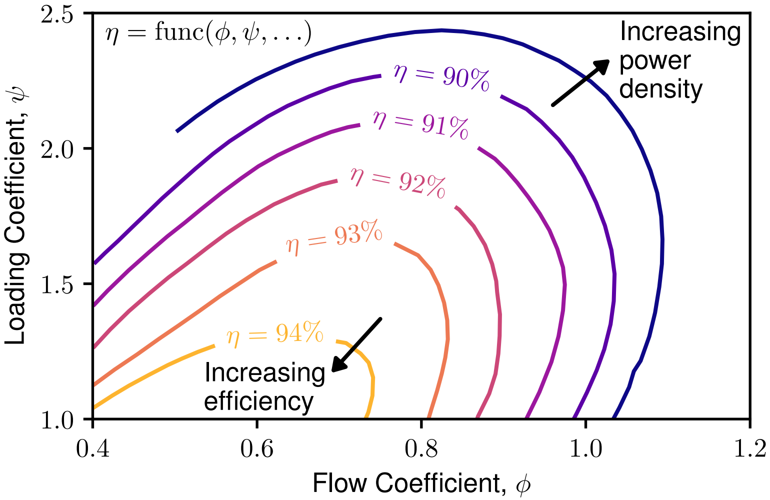
Preliminary design locks in performance



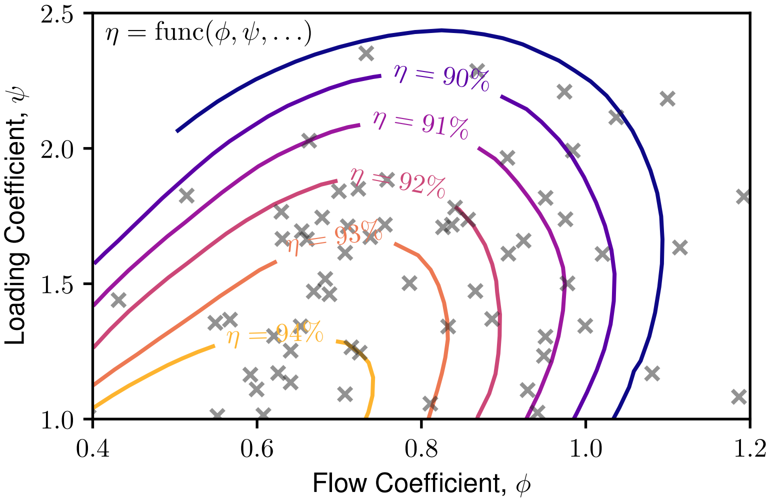
Most important early decisions are informed by lowest-fidelity methods

Compromises in preliminary design — Smith chart



Smith, S.F. (1965). "A Simple Correlation of Turbine Efficiency."
Aeronaut. J. Vol. 69, No. 655, p. 467–470.

Compromises in preliminary design — Smith chart



Smith, S.F. (1965). "A Simple Correlation of Turbine Efficiency."
Aeronaut. J. Vol. 69, No. 655, p. 467–470.

In this talk

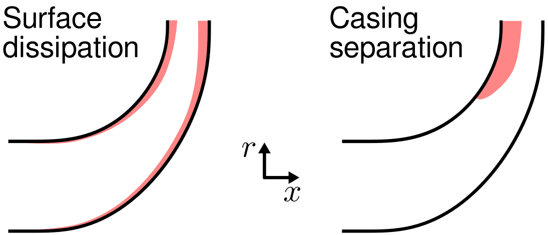
New tools for design space mapping

$$\eta = \mathrm{func}\big(PR_{\mathrm{tt}},\ \phi_1,\ Ma_1^{\mathrm{rel}},\ HTR_1,\ DH,\ \alpha_2^{\mathrm{rel}},\ C_\Gamma,\ \tau\big)$$

fitted to RANS solutions of
3700 radial compressors
spanning the 8D design space

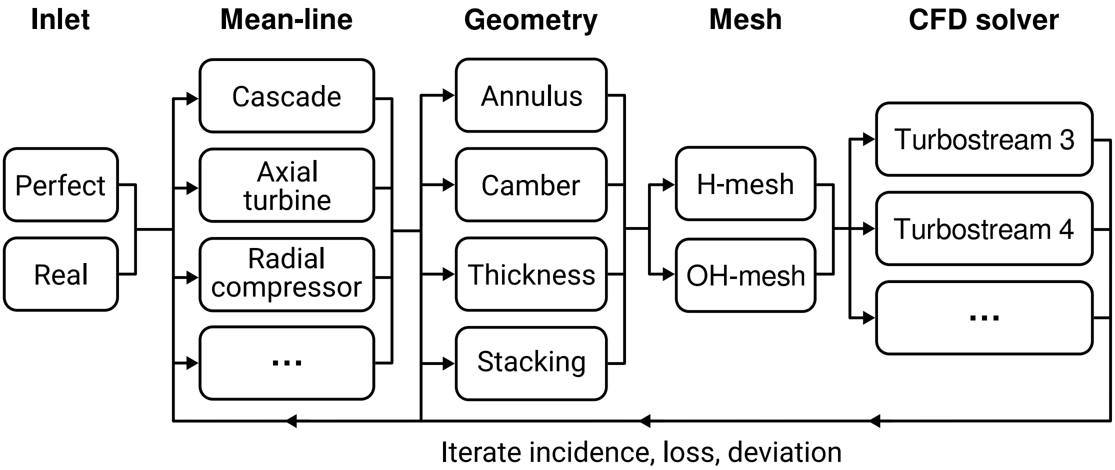
New physical understanding

Two mechanisms drive efficiency trends



Constructing a radial compressor design space map

TURBIGEN open-source design system

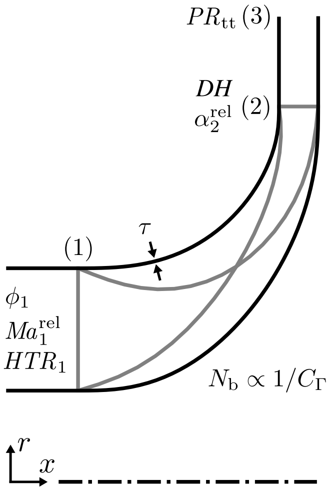


Radial compressor defined by eight design variables

Design variable		
Total pressure ratio	PR_{tt}	$= p_{03}/p_{01}$
Inlet flow coefficient	ϕ_1	$= V_{x1}/U_1$
Inlet relative Mach	Ma_1^{rel}	$= V_1^{\mathrm{rel}}/a_1$
Inlet hub-to-tip ratio	HTR_1	$= r_{\mathrm{h1}}/r_{\mathrm{t1}}$
Rotor de Haller	DH	$= V_2^{\mathrm{rel}}/V_1^{\mathrm{rel}}$
Exit relative yaw angle	$\alpha_2^{\mathrm{rel}}/°$	
Circulation coefficient	C_Γ	$= \oint V^{\mathrm{rel}}\mathrm{d}\ell / V_1^{\mathrm{rel}}\ell$
Tip clearance/span	$\tau/\%$	

Vaneless diffuser $V_3/V_2 = 0.75$

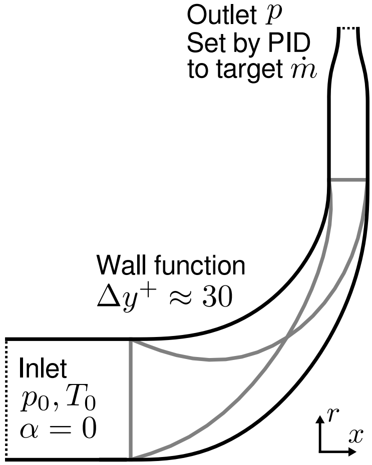
$\alpha_1 = 0°$, $Ma_1^{\mathrm{rel}} < 1$, $Re_\ell = 5 \times 10^6$



RANS computations in Turbostream 3

- Compressible RANS solver, 2nd-order accurate
- Spalart-Allmaras turbulence model
- H-mesh topology $\sim 2 \times 10^6$ nodes
- One simulation $\sim$ 10 min on A100 GPU
- Iterative loss/deviation correction $\sim$ 1 hour

Note: `TURBIGEN` CFD solver is interchangeable



Take 3700 random samples over 8D design space

Design variable		Datum	Lower	Upper
Total-to-total pressure ratio	PR_{tt}	2.0	1.5	3.5
Inlet flow coefficient	ϕ_1	0.6	0.35	1.0
Inlet relative Mach	Ma_1^{rel}	0.6	0.3	0.9
Inlet hub-to-tip ratio	HTR_1	0.5	0.2	0.8
Rotor de Haller	DH	1.0	0.6	1.4
Exit relative yaw angle	$\alpha_2^{\mathrm{rel}}/^{\circ}$	-60	-80	-20
Circulation coefficient	C_{Γ}	0.6	0.4	1.0
Tip clearance span fraction	$\tau/\%$	1.0	0.5	5.0

Latin hypercube — one sample in each equally probable row and column

Polynomial regression to fit the design space

$$\eta_{\mathrm{tt}}(\mathbf{x}) = \sum_j c_j \prod_i P_{k_{ij}}(x_i) \quad \text{where}$$

x_i = vector of independent variables

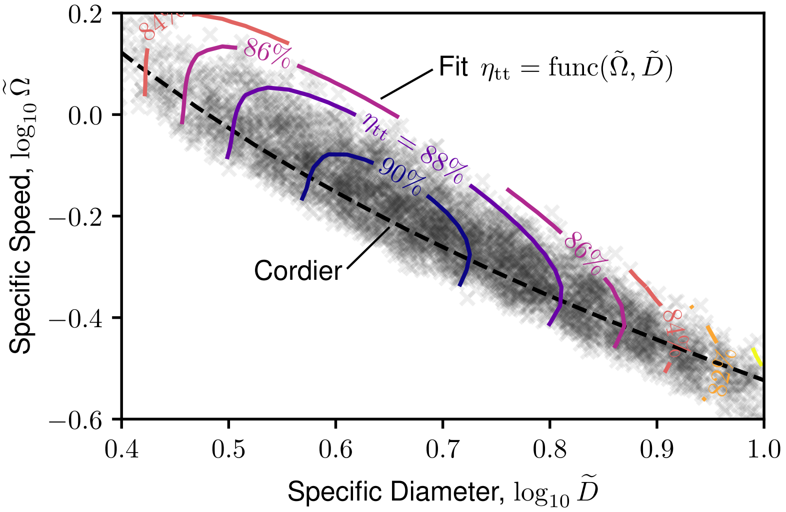
c_j = coefficient vector least-squares fitted to data

P_k = Legendre polynomial of order k

k_{ij} = matrix of polynomial orders, all combinations summing $\leq k_{\max}$

$k_{\max} = 3$ yields a root-mean-square test error of 1.2% η_{tt}

Fitted efficiency is consistent with Cordier line



$$\tilde{\Omega} = \Omega \frac{(\dot{m}/\rho_1)^{0.5}}{(\Delta h_0)^{0.75}}$$

$$\tilde{D} = 2r_3 \frac{(\Delta h_0)^{0.25}}{(\dot{m}/\rho_1)^{0.5}}$$

Cordier, O. (1953). “Ähnlichkeitsbedingungen für Strömungsmaschinen.”
BWK Bd Vol. 6(10) pp. 337-340

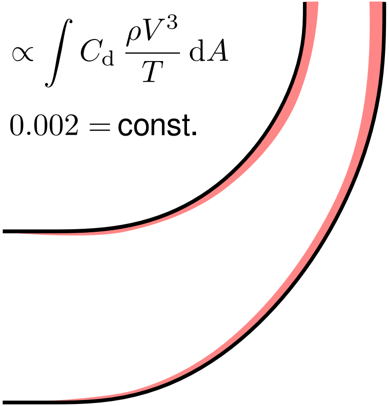
Loss mechanisms governing efficiency trends

Efficiency metrics for loss mechanisms

Surface dissipation in boundary layers

$$\Delta\eta_{\mathrm{srf}} \propto \int C_{\mathrm{d}} \frac{\rho V^3}{T}\,\mathrm{d}A$$

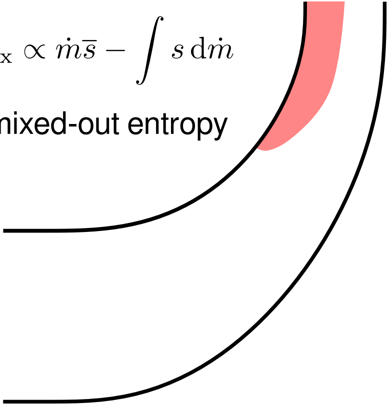
$C_{\mathrm{d}} \approx 0.002 =$ const.



Mixing loss in casing separations

$$\Delta\eta_{\mathrm{mix}} \propto \dot{m}\overline{s} - \int s\,\mathrm{d}\dot{m}$$

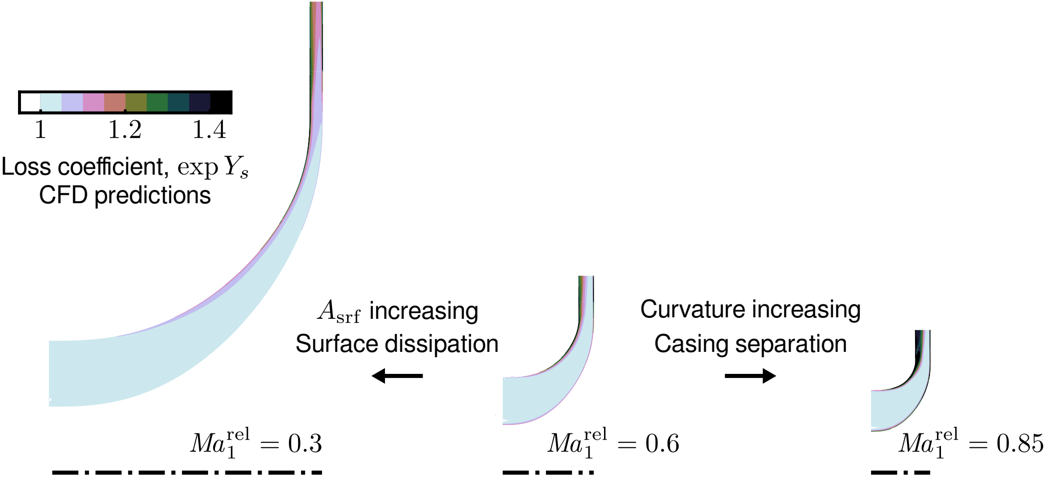
$\overline{s} =$ mixed-out entropy



Methodology for loss mechanism study

- Calculate 8D polynomial fits for $\Delta\eta_{\mathrm{tt}}$, $\Delta\eta_{\mathrm{srf}}$, and $\Delta\eta_{\mathrm{mix}}$
- Start with a datum compressor at the centre of the design space
- Vary each design variable in turn, holding others constant
- Plot *changes* in *lost* efficiency with respect to the datum compressor
- Trends in approximate metrics $\Delta\eta_{\mathrm{srf}}$ and $\Delta\eta_{\mathrm{mix}}$ explain trends in actual $\Delta\eta_{\mathrm{tt}}$

Effect of inlet Mach number on loss



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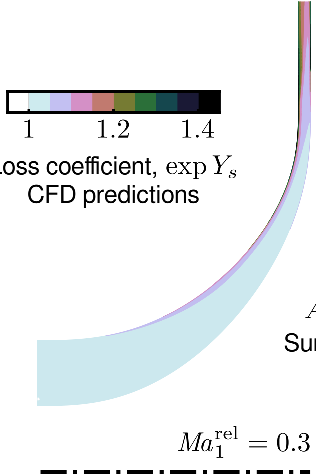
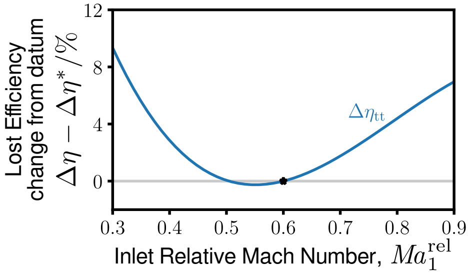
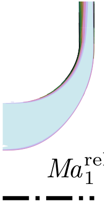


Loss coefficient, $\exp Y_s$
CFD predictions



$Ma_1^{rel} = 0.3$

A_{srf} increasing
Surface dissipation
←

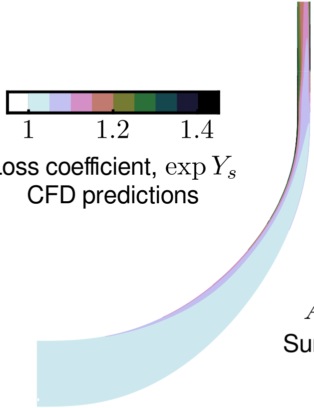


$Ma_1^{rel} = 0.6$

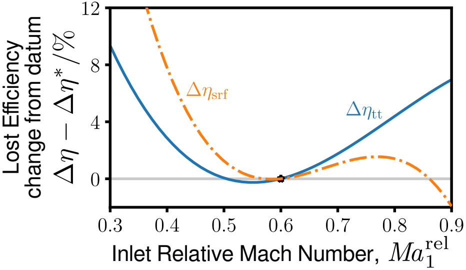
Curvature increasing
Casing separation
→

$Ma_1^{rel} = 0.85$

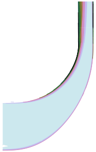
Effect of inlet Mach number on loss



Loss coefficient, $\exp Y_s$
CFD predictions



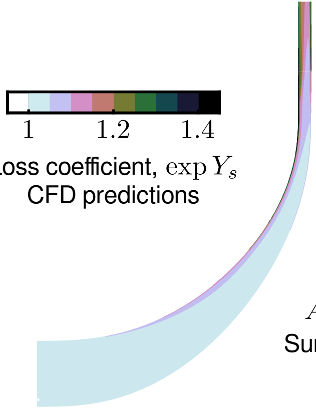
A_{srf} increasing
Surface dissipation
←



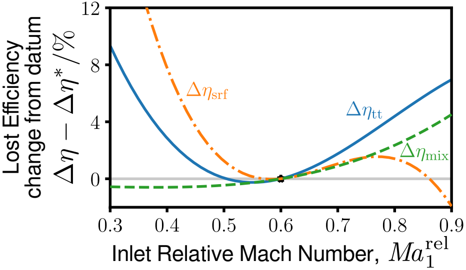
Curvature increasing
Casing separation
→

$Ma_1^{\mathrm{rel}} = 0.3$ $\quad$ $Ma_1^{\mathrm{rel}} = 0.6$ $\quad$ $Ma_1^{\mathrm{rel}} = 0.85$

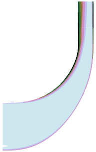
Effect of inlet Mach number on loss



Loss coefficient, $\exp Y_s$
CFD predictions



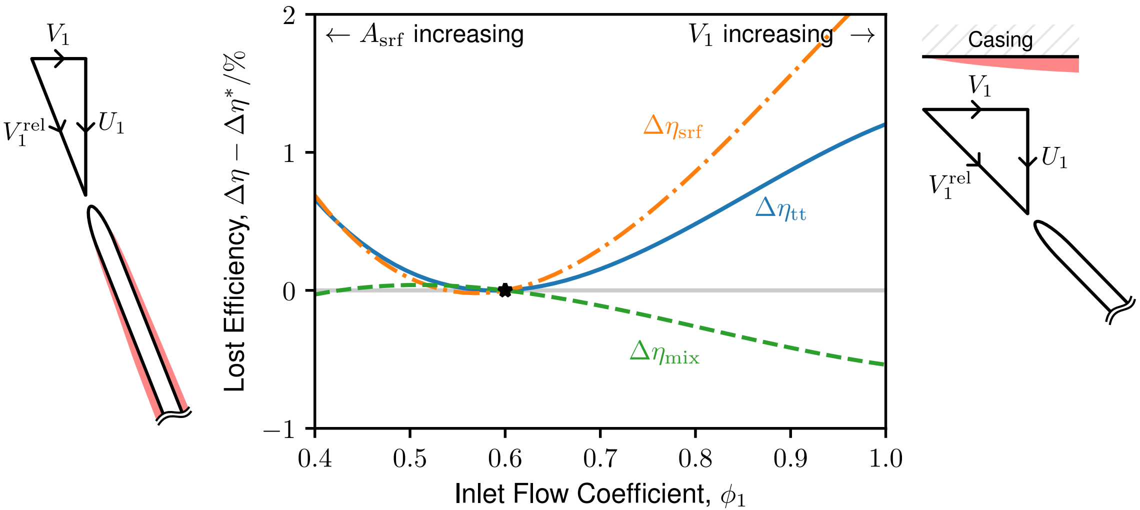
A_{srf} increasing
Surface dissipation



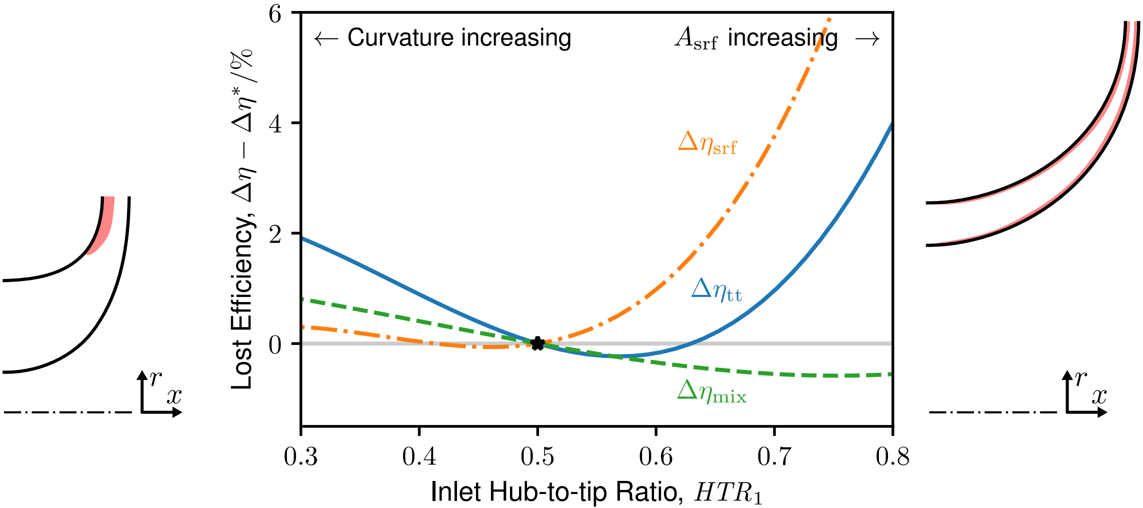
Curvature increasing
Casing separation

$Ma_1^{\mathrm{rel}} = 0.3$ $\quad$ $Ma_1^{\mathrm{rel}} = 0.6$ $\quad$ $Ma_1^{\mathrm{rel}} = 0.85$

Flow coefficient primarily affects surface dissipation



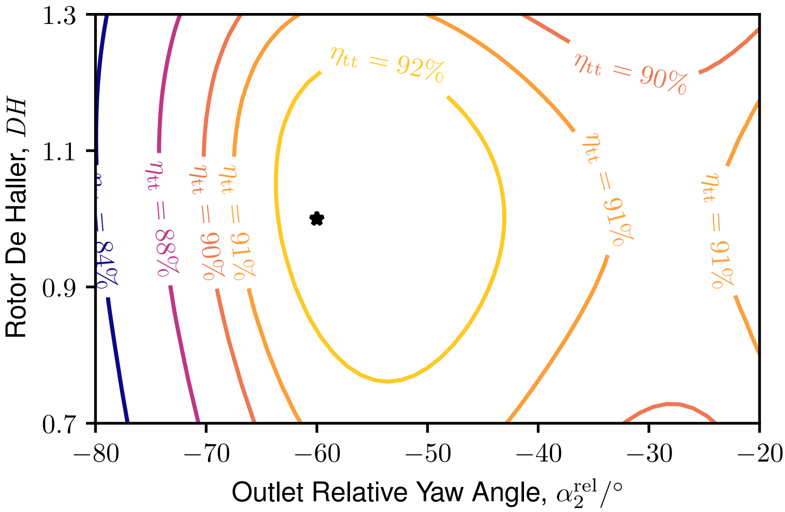
Surface dissipation dominates at high hub-to-tip ratio



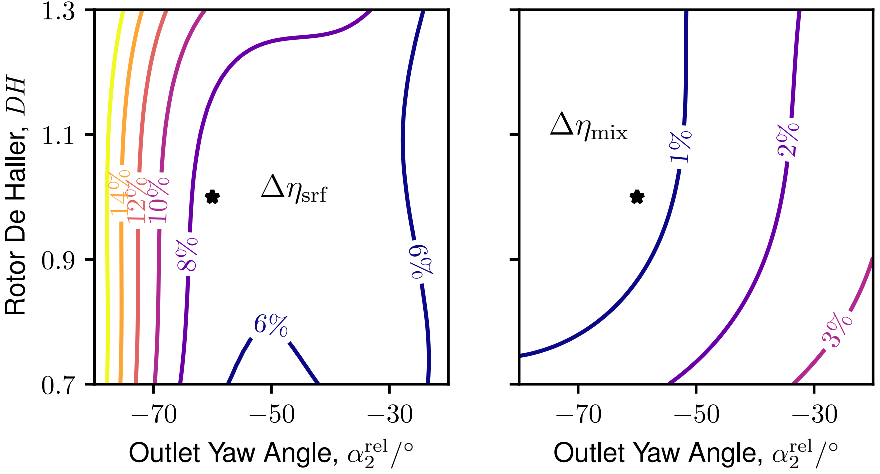
Design space boundaries for inlet variables

Design variable		Lower	Upper
Inlet flow coefficient	ϕ_1	surf	surf
Inlet relative Mach	Ma_1^{rel}	surf	mix
Inlet hub-to-tip ratio	HTR_1	mix	surf
Rotor de Haller	DH		
Exit relative yaw angle	$\alpha_2^{\mathrm{rel}}/^\circ$		
Circulation coefficient	C_Γ		

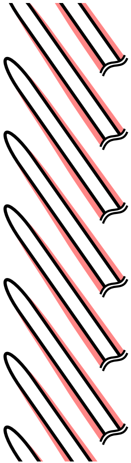
Efficiency as function of exit velocity triangle

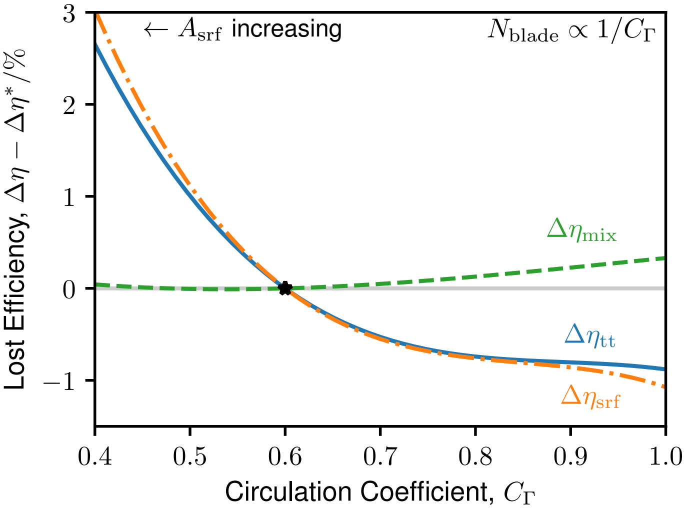


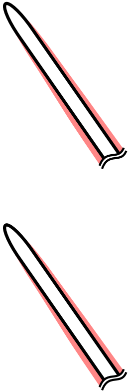
Surface dissipation and mixing losses set boundaries



Surface dissipation captures number of blades effect







Design space boundaries for outlet variables

Design variable		Lower	Upper
Inlet flow coefficient	ϕ_1	surf	surf
Inlet relative Mach	Ma_1^{rel}	surf	mix
Inlet hub-to-tip ratio	HTR_1	mix	surf
Rotor de Haller	DH	mix	surf
Exit relative yaw angle	$\alpha_2^{\mathrm{rel}}/^\circ$	surf	mix
Circulation coefficient	C_Γ	surf	mix

Conclusions

- Large ensembles of automated designs and simulations provide a higher-fidelity replacement for legacy empirical correlations in preliminary design
- CFD-based design space map for 3700 radial compressors is consistent with the Cordier line; 8D polynomial surface fit yields a test RMS error of 1.2% efficiency
- Aerodynamic optimum mean-line design is set by a balance of surface dissipation in boundary layers and mixing losses in casing separations

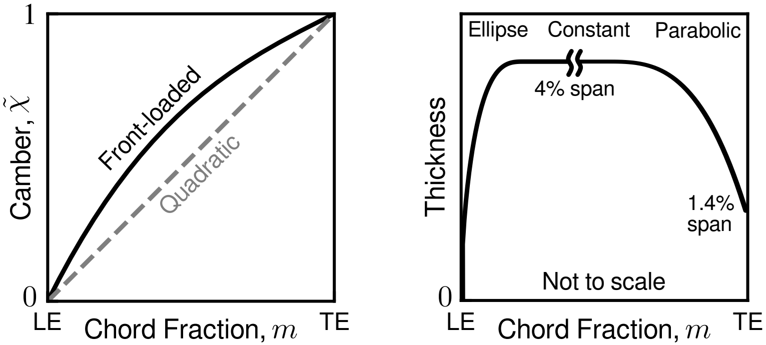
Interactive radial compressor designer at `https://whittle.digital/rcd`

TURBIGEN code, documentation and examples at `https://turbigen.org`

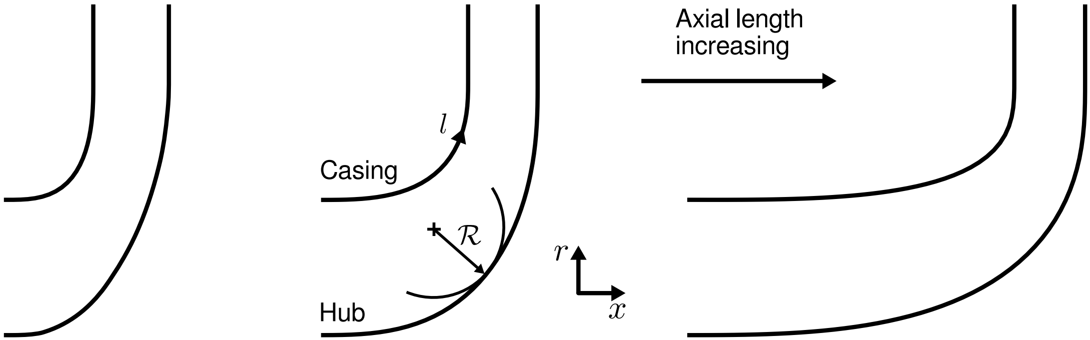
Appendix

Curvature-continous splines for camber and thickness

$$\tilde{\chi}(m) = \frac{\tan\chi(m) - \tan\chi_1}{\tan\chi_2 - \tan\chi_1}$$



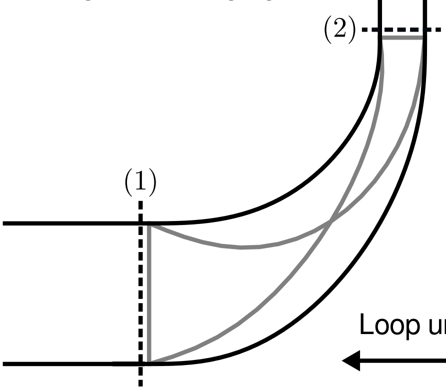
Optimise axial length to minimise integrated curvature



Minimise $\displaystyle\int_{\text{hub}} \kappa^2 \,\mathrm{d}l + \int_{\text{casing}} \kappa^2 \,\mathrm{d}l$ where curvature $\kappa = \dfrac{1}{\mathcal{R}}$

Iterative post-processing to match design intent

Unstructured cuts at
leading and trailing egdes



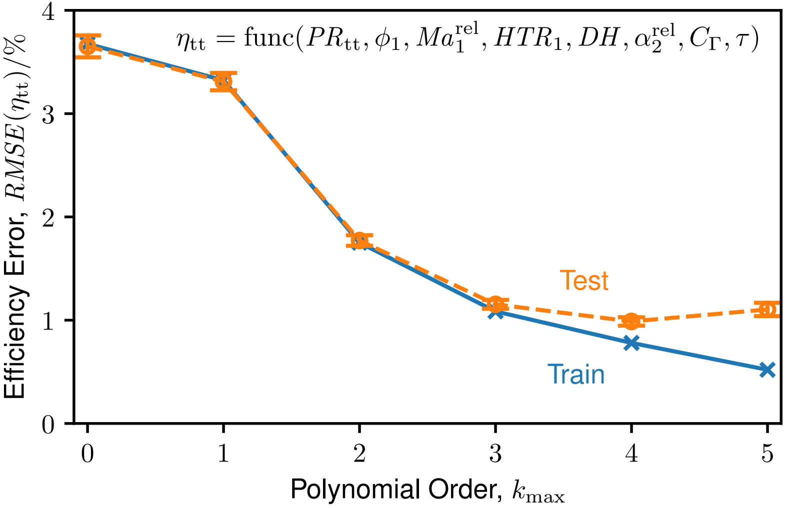
Mix out at constant area, conserving:
mass, momentum, and energy

Update mean-line efficiency guess
Recamber χ_1 for zero incidence
Recamber χ_2 to match design α_2

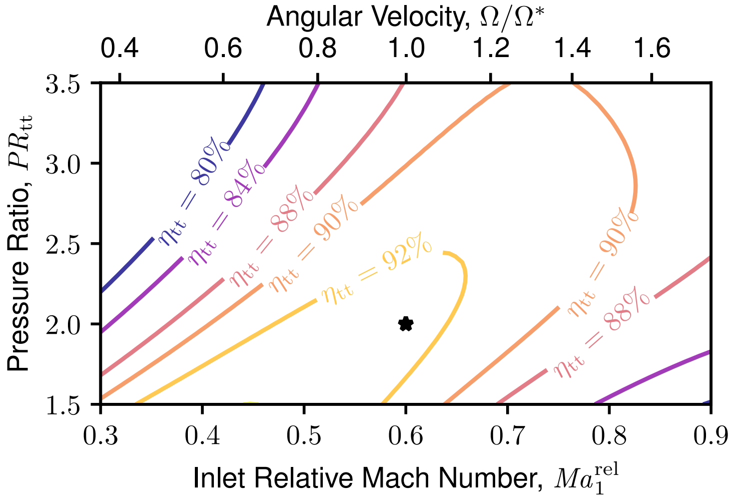
Create new geometry
Run CFD solver

Loop until converged

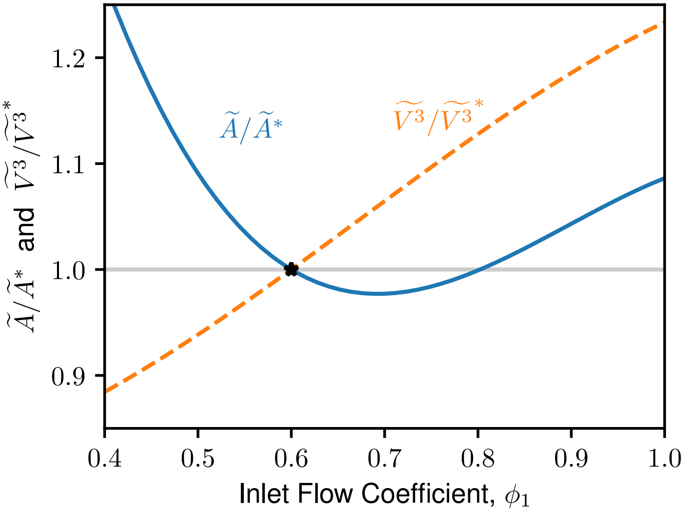
Crossvalidation to select 8D polynomial order



Optimum Mach number rises with pressure ratio



Flow coefficient sets area–velocity tradeoff



Surface area made non-dimensional with duty

$$\widetilde{A} = A_{\mathrm{srf}} \frac{\rho_1 \sqrt{\Delta h_0}}{\dot{m}}$$

Lost efficiency per unit non-dimensional area

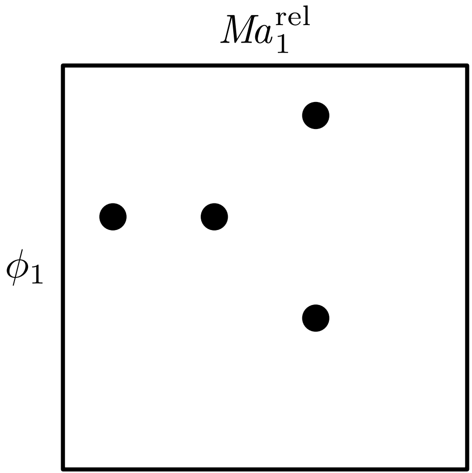
$$\widetilde{V^3} = \frac{\Delta \eta_{\mathrm{srf}}}{\widetilde{A}}$$

Kaufmann, S.M. (2020). *A Unified Turbine Preliminary Design Study*
PhD thesis, University of Cambridge.

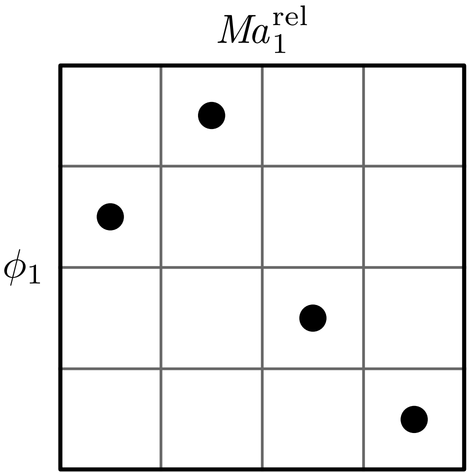
Ranking of design variables by peak-to-peak efficiency

Design variable		η_{tt} range	Low	High
Exit relative yaw angle	$\alpha_2^{\mathrm{rel}}/^\circ$	8.4%	srf	mix
Inlet relative Mach	Ma_1^{rel}	7.7%	srf	mix
Inlet hub-to-tip ratio	HTR_1	4.6%	mix	srf
Tip clearance	τ	3.8%	—	—
Circulation coefficient	C_Γ	3.5%	srf	mix
Rotor de Haller	DH	3.0%	mix	srf
Inlet flow coefficient	ϕ_1	1.3%	srf	srf

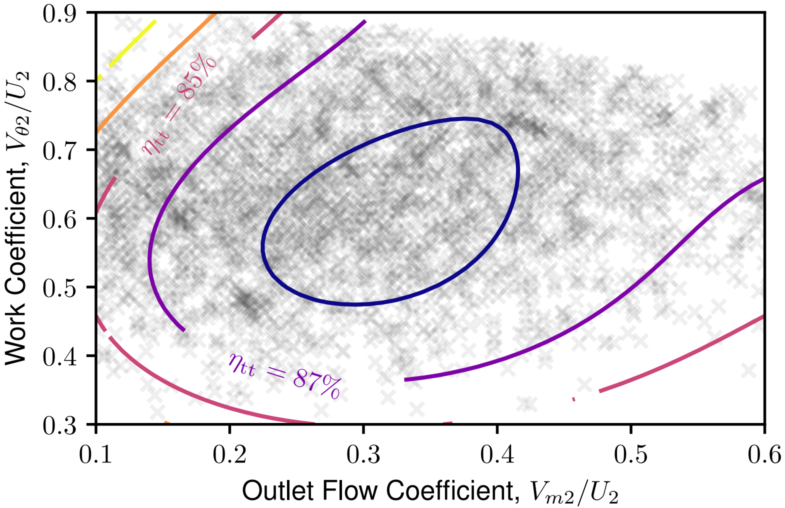
Random sampling



Latin hypercube sampling



Comparison to Casey and Robinson (2023)



Whittle Laboratory

Casey, M. and Robinson, C. (2023). "Some Properties of the Exit Velocity Triangle of a Radial Compressor Impeller". *J. Turbomach.* 145(5)