Consolidating data for boundary layer transition onset under the influence of free stream turbulence and pressure gradients

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Laminar-turbulent transition in boundary layers is important to turbomachinery flows. Accurate prediction of boundary layer state is necessary when calculating entropy generation and heat transfer rates on blades and end walls. Current practice uses a mix of empirical correlations, experimental testing and the experience of the designer to account for transition; however, in complex flows at the decreasing Reynolds numbers found in modern machines, prior knowledge is insufficient. As such, there remains a need for correlations that can be fed into low-order methods for routine use in design. Many such correlations are available; however, the data they are based on is limited, particularly for flows with strong pressure gradients and high free stream turbulence such as those found in turbomachinery. This paper consolidates existing literature data for boundary layer transition onset under the influence of free stream turbulence and pressure gradients. Details of the methods used in the original studies are discussed. Data is compared to existing correlations and areas with insufficient data are identified, along with suggestions for future work to fill gaps in the literature.

Keywords: boundary layer, transition, turbulence, pressure gradient

1 Introduction

The purpose of this paper is to consolidate literature data for boundary layer transition onset, providing a single resource to the community for improving existing models and developing new ones. It is hoped that this will encourage investigators to provide further data where it is needed by reporting results with turbulence, pressure gradient and transition Reynolds number measurements.

In his 1991 review of the role of laminar-turbulent transition in gas turbines, Mayle [1] concluded that more experimental data is needed to develop models for flows subject to high free stream turbulence and strong pressure gradients. Routine use of CFD in design means it is vital that simulations are able to capture transitional behaviour in turbomachinery flows. In their recent review of transition modelling Dick and Kubacki [2] found that no available model fully captures transitional behaviour; however, Dick and Kubacki [2] found that the model of Menter et al. [3], which uses an empirical correlation for transition onset, performs well in many, but not all, cases. Similarly, the widely used codes MISES [4] and TEXSTAN [5] use versions of the Abu-Ghannam and Shaw [6] correlation for predicting transition. These correlations are based on experimental data that is sparse in many areas of interest. Turbulence intensity in turbomachinery may be as high as 20 % [7] while Fig. 1 shows that Thwaites' pressure gradient parameter can be as high as 0.11 at certain points on an aerofoil. For flows with pressure gradients, Menter et al. [3] and Abu-Ghannam and Shaw [6] compare their correlations to limited data sets with values of turbulence intensity below 5 % and Thwaites' parameter below 0.075, so the resulting correlations are forced to extrapolate beyond experimental data when applied to real flows. Improving, developing and validating models requires larger data sets that give complete coverage of the design space.

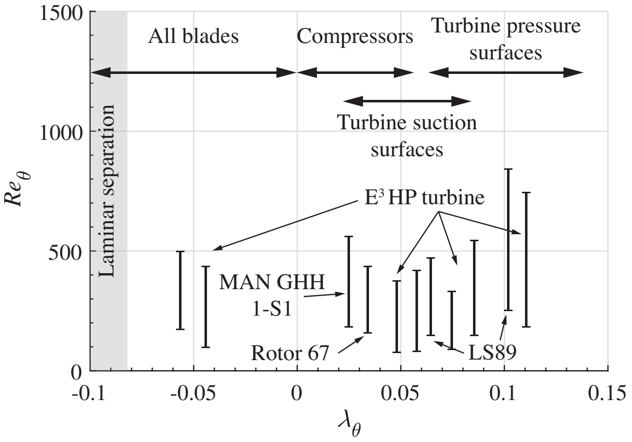

Fig. 1 Typical values of Thwaites' parameter and momentum thickness Reynolds number calculated from data on selected turbomachinery test cases [8–11]. Each line spans a chord Reynolds number range of 1×10^5-2×10^6.

2 Transition Data

This paper considers data collected for attached flow transition affected by turbulence intensity and pressure gradients. Many secondary parameters have been suggested to affect transition. These include, but are not limited to: surface roughness, boundary layer skew, Mach number and surface curvature. Each of these parameters affect transition differently and for some there is minimal data available to assess. This work therefore focuses solely on turbulence intensity and pressure gradient to minimise the dimensionality of the data set. Transition in separated flows and transition caused by unsteady wake passing both involve mechanisms that are

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Table 1 Transition onset data sources

Source	Year	Geometry	Method	Tu location	L reported?
Dryden [12]	1937	Flat plate	Hot-wire	N/A	No
Hislop [13]	1940	Flat plate	Hot-wire	N/A	Yes
Fage and Preston [14]	1941	Body of revolution	Ink visualisation	Transition point	No
Liepmann [15]	1943	Flat and curved plates	Hot-wire and pitot tube	N/A	No
Schubauer and Skramstad [16]	1948	Flat plate	Pitot tube	Leading edge	Yes
Bennett [17]	1953	Flat plate	Hot-wire	N/A	Yes
Feindt [18]	1956	Flat plate	Hot-wire	N/A	No
Wells [19]	1967	Boundary layer channel	Pitot tube	N/A	Yes
Hall [20]	1968	Flat plate	Pitot tube	Average	No
Brown and Burton [21]	1978	Aerofoil	Heat transfer	N/A	Yes
Martin et al. [22]	1978	Cascade	Heat transfer	Leading edge	No
Abu-Ghannam and Shaw [6]	1980	Flat plate	Hot-wire	Midway from LE	Yes
Blair [23]	1982	Flat plate	Heat transfer	Average	Yes
Gostelow and Blunden [24]	1989	Flat plate	Hot-wire	Leading edge	No
Sohn and Reshotko [25]	1991	Flat plate	Hot-wire	Average	Yes
Fasihfar and Johnson [26]	1992	Flat plate	Hot-wire	Transition point	No
Savill [27]	1993	Flat plate	Hot-wire	Leading edge	Yes
Gostelow et al. [28]	1994	Flat plate	Hot-wire	Leading edge	No
Roberts and Yaras [29]	2003	Flat plate	Hot-wire	Transition point	Yes

unlikely to be adequately captured by an attached flow correlation, so the data set presented here is exclusively for attached flows.

Data is collated for three parameters: momentum thickness Reynolds number at transition $Re_{\theta,\mathrm{t}}$, free stream turbulence intensity Tu and Thwaites' parameter λ_θ. These are defined as:

$$Re_{\theta,\mathrm{t}} = \frac{U\theta}{\nu} \quad (1) \qquad Tu = \frac{u'}{U} \quad (2) \qquad \lambda_\theta = \frac{\theta^2}{\nu}\frac{dU}{dx} \quad (3)$$

where U is the boundary layer edge velocity, u' is the turbulent fluctuation velocity, θ is the boundary layer momentum thickness, ν is the kinematic viscosity of the fluid and x is the stream-wise coordinate. The effect of pressure gradient on boundary layers is often investigated using an acceleration parameter K given by

$$K = \frac{\nu}{U^2}\frac{dU}{dx} \quad (4)$$

Mayle [1] suggests that K is a more useful parameter for describing bypass transition, however almost all literature data is given in terms of Thwaites' parameter, so λ_θ is used here. Data can be readily transformed using the identity $K = \lambda_\theta / Re_\theta^2$.

2.1 Data sources. The sources for the data sets consolidated in this work are shown in Table 1. In all, 19 sources are included spanning almost 70 years of research. Exact experimental methods can be found in each source, pertinent details are presented here:

- Most data is taken on flat plates at low Mach numbers ($<<0.3$), aside from the data from Martin et al. [22] which is at a Mach number just below 1. All experiments are in air apart from the water tunnel data of Fage and Preston [14].

- Although turbulence intensity magnitude is required, the location at which it is measured is often not reported, or reported in unclear reference to the test section. Most studies appear to measure Tu at the inlet of the working section only, or use an average Tu over the whole section. Only three studies explicitly measure Tu at the location of transition, while data from Abu-Ghannam and Shaw [6] is given with Tu measured midway between the transition location and the leading edge of the plate. The issue of inconsistent Tu reference location was raised by Dick and Kubacki [2] as a limitation of existing models. Praisner and Clark [30] discussed the importance of using local values of Tu, meaning inlet turbulence is likely to give the wrong answer in flows with pressure gradients. Some remedies have been proposed such as the correction for inlet-to-local Tu from Steelant [31] which Clark et al. [32] found to assist in aligning data with correlations. This is a major sticking point that future work should look to address.

- Additional properties of the free stream turbulence are inconsistently reported. In some studies it is noted that the turbulence is not fully isotropic [15,16], while in others isotropy is not reported. The length scale, L, or spectral properties of turbulence are reported in about half of the studies considered, although sometimes only by reference to the dimensions of the turbulence grid [13,21]. Praisner and Clark [30] and Jonáš et al. [33] both demonstrated that turbulence length scale plays a role in transition, and physical reasoning suggests that turbulence should be on the scale of the boundary layer in order to influence it, however more work is needed on this effect.

- Multiple methods for establishing the location of transition onset are used. Hot-wires are the most common method, with transition being detected by the onset of intermittency in the laminar boundary layer. Fage and Preston [14] took a similar measurement with ink flow visualisation in their water tunnel. Pitot-static measurements are also used to detect a rise in dynamic pressure close to the wall as the boundary layer transitions. Both hot-wires and flattened pitot probes can be used to measure velocity profiles with a boundary layer traverse. Transition is then detected from changes in integral parameters. Finally, heat transfer measurements on heated or cooled walls allow for transition detection from an increase in heat transfer coefficient. Many of these methods require time consuming traverses to detect transition and were limited by technology available at the time. Modern experimentalists may look to use flow visualisation techniques such as infrared thermography [34] to detect transition in real time.

There is considerably more data available from zero pressure gradient experiments. This is to be expected as it removes a dimension from the problem. Some zero pressure gradient results and individual transition measurements on blades and aerofoils found in the literature have not been included as they are not reported in compatible parameters [35,36]. In addition to this some sources cited by other authors have not been included as results could not be traced to the original source. In particular the data set reported by Praisner and Clark [30] has not been included as

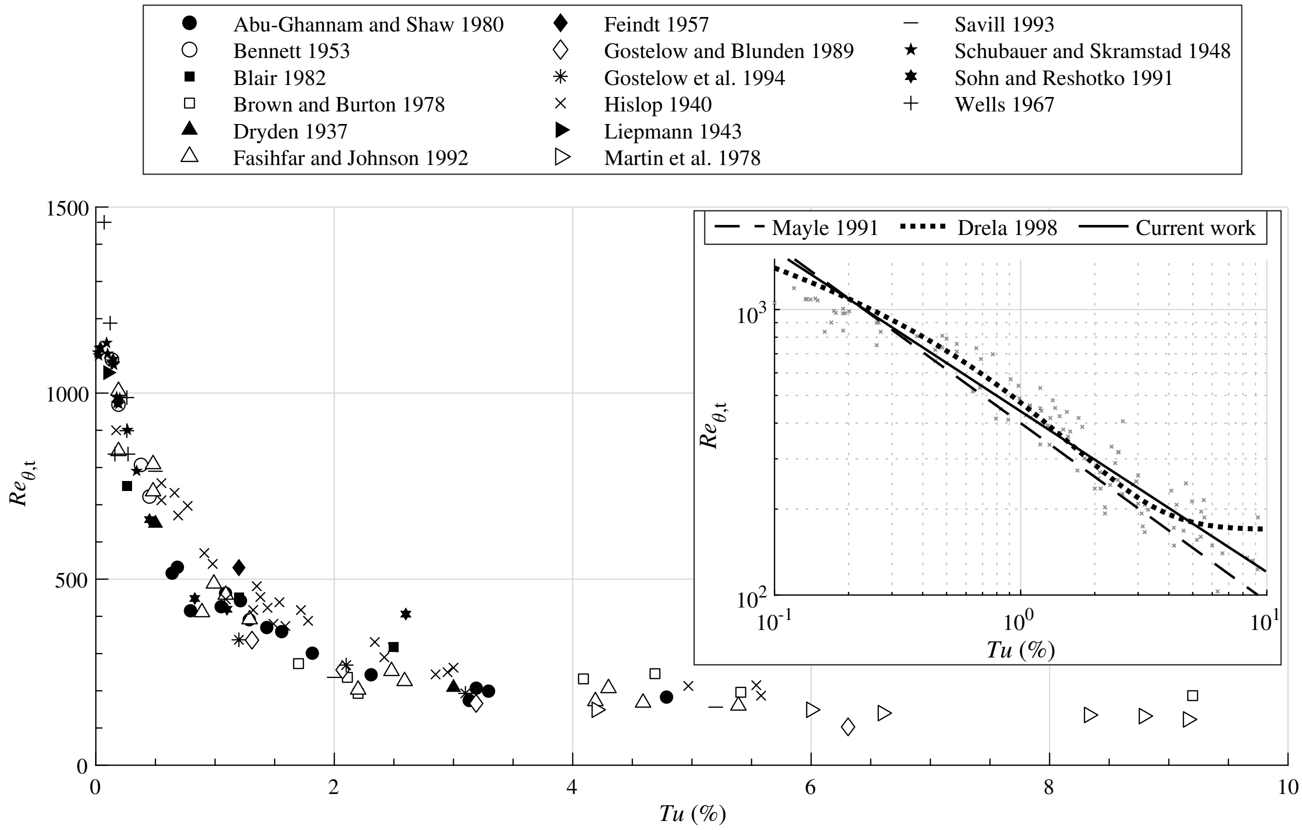

Fig. 2 Zero pressure gradient transition data labelled by source. Data plotted with correlations inset.

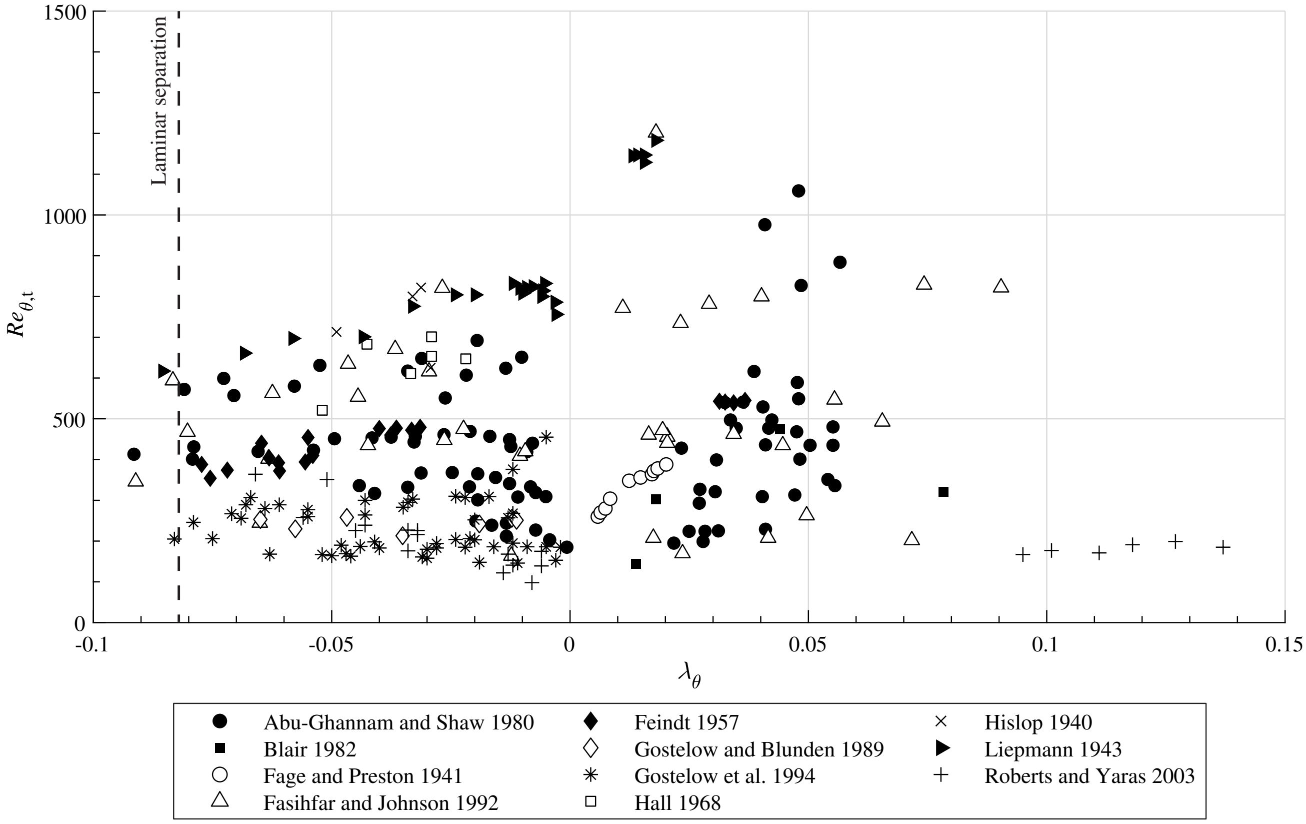

Fig. 3 Pressure gradient transition data labelled by source. Symbols are matched to Fig. 2 where appropriate.

it is from a proprietary cascade data base for which there is no additional information.

The complete data set with sources labelled is shown in Figs. 2 and 3, plotted as $Re_{\theta,\mathrm{t}}$ against Tu, for zero pressure gradient, and λ_θ, for non-zero pressure gradient. There are 382 data points in total from 19 sources.

2.2 Correlations. Many authors have developed correlations for $Re_{\theta,\mathrm{t}}$ against Tu and λ_θ. While assessing these correlations is not the focus of this paper they are mentioned here for completeness. Hourmouziadis [37] and Mayle [1] both proposed correlations for transition in a zero pressure gradient flow. These correlations are both of the form $Re_{\theta,\mathrm{t}} = cTu^n$, with $c \approx 400$ and $n \approx -0.6$ found to agree well with experimental data. For flows with pressure gradients, several authors have constructed more complex models based on empirical data and analytic methods. The models of van Driest and Blumer [38], Seyb [39], Fasihfar and Johnson [26] and Suzen et al. [40] make minimal attempt to differentiate between adverse and favourable pressure gradients, and therefore tend to under-predict the effects of both. Hall and Gibbings [41] and Dunham [42] construct models that do have significantly different behaviour between adverse and favourable pressure gradients, however they tend to dramatically over-predict the effect of acceleration in delaying transition. The correlations of Abu-Ghannam and Shaw [6] and Langtry and Menter [43] are the most widely used in modern practice. These apply separate functions for adverse and favourable pressure gradients, resulting in a non-smooth function at $\lambda_\theta = 0$. These correlations better capture transitional behaviour in both adverse and favourable pressure gradients; however, in doing so they sacrifice accuracy in zero pressure gradient flows. for which there is considerably more data. Langtry and Menter's [43] correlation over-predicts $Re_{\theta,\mathrm{t}}$ at low values of Tu while under-predicting at high Tu. The Abu-Ghannam and Shaw [6] correlation reaches a limit of $Re_{\theta,\mathrm{t}} = 163$ at high Tu, which is both physically unsatisfying and not seen in experimental data.

3 Zero Pressure Gradient Flows

In this section, data from zero pressure gradient measurements is discussed separately from data with pressure gradients. The zero pressure gradient data set is plotted in Fig. 2 along with three correlations. Mayle's [1] correlation is given by

$$Re_{\theta,\mathrm{t}} = \frac{400}{Tu^{0.625}} \qquad (5)$$

A new correlation has been developed in the current work, which takes the the same form as Eq. 5 but is calculated from a least squares fit to the full data set shown in Fig. 2. This correlation is

$$Re_{\theta,\mathrm{t}} = \frac{440}{Tu^{0.56}} \qquad (6)$$

It can be seen from Eqs. 5 and 6, and from Fig. 2, that these two correlations do not differ significantly. Mayle's correlation tends to sit on the lower bound of the data set, while the new correlation predicts transition at higher values of $Re_{\theta,\mathrm{t}}$, particularly at higher levels of turbulence. Both correlations capture the trends in the data well, and any correlation that includes non-zero pressure gradients should seek to match these correlations for zero pressure gradient.

It is notable that neither correlation agrees as well for data with turbulence intensity less than 2 %. A correlation of this form makes no attempt to differentiate between the two common modes of attached flow transition: "natural" transition via Tollmien-Schlichting waves, predicted by linear stability theory at low turbulence, and "bypass" transition caused by large disturbances that bypass the amplification of T-S waves. In adapting the Abu-Ghannam and Shaw [6] correlation for use in MISES, Drela [44] attempted to blend the correlation with linear stability theory using Mack's [45] correlation for amplification factor. Drela's [44] blended correlation is also plotted in Fig. 2, where it can be seen to better follow the data for $Tu < 2$ %, but due to the limiting behaviour of the Abu-Ghannam and Shaw [6] correlation it overestimates $Re_{\theta,\mathrm{t}}$ at higher turbulence intensities.

4 Flows with Pressure Gradients

In this section, data from transition measurements *with* pressure gradients is discussed. The pressure gradient data set is shown in Fig. 3. Data is limited to pressure gradients in the range $-0.1 < \lambda_\theta < 0.15$. Generating strong favourable pressure gradients with transition is difficult in experiments, with only the study from Roberts and Yaras [29] reporting measurements for $\lambda_\theta > 0.1$. In adverse pressure gradients measurements should be limited to values of λ_θ greater than -0.082 by the Thwaites criterion for laminar separation [46]. In Fig. 3 there are five data points with $\lambda_\theta < -0.082$. Three of these are within measurement error, however two points from Abu-Ghannam and Shaw [6] and Fasihfar and Johnson [26] are at $\lambda_\theta < -0.09$ and should be used with caution.

Figure 4 re-plots the full data set, including zero pressure gradient data, with data points coloured by turbulence intensity. Overall there is good agreement across the data sets, however there is significant scatter. The most notable outlier is a set of data points with $Tu < 0.5$ % and $\lambda_\theta < 0$ that show transition occurring at $Re_{\theta,\mathrm{t}} < 500$, while all other points with similar turbulence intensity indicate $Re_{\theta,\mathrm{t}} > 500$. This data is from the study by Gostelow et al. [28]. The original authors noted that their low turbulence data showed significant disagreement with other sources, however no further explanation was given.

Also plotted in Fig. 4 are the correlations of Abu-Ghannam and Shaw [6] and Langtry and Menter [43], with lines coloured by Tu in the same way as the data. For flows with pressure gradients, Menter et al. [3] compare their correlation to data from Fasihfar and Johnson [14] while Abu-Ghannam and Shaw compare to their own data as well as that from Refs. [13–15,18,20]. Both correlations capture the effect of adverse pressure gradients causing earlier transition, particularly at low turbulence levels, and show a weaker effect of favourable pressure gradients delaying transition. For $Tu < 2$ % the correlations suggest that pressure gradient has a negligible effect on transition.

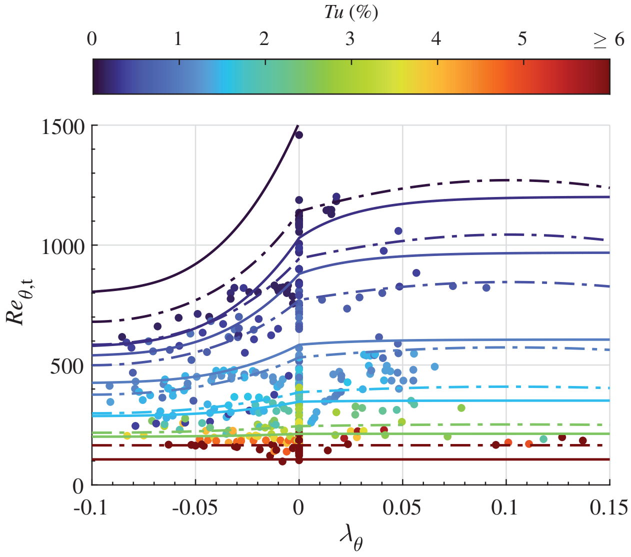

Fig. 4 Pressure gradient transition data coloured by *Tu*. Correlations from Abu-Ghannam and Shaw [6] (dashed) and Langtry and Menter [43] (solid) are plotted with lines coloured to the same scale.

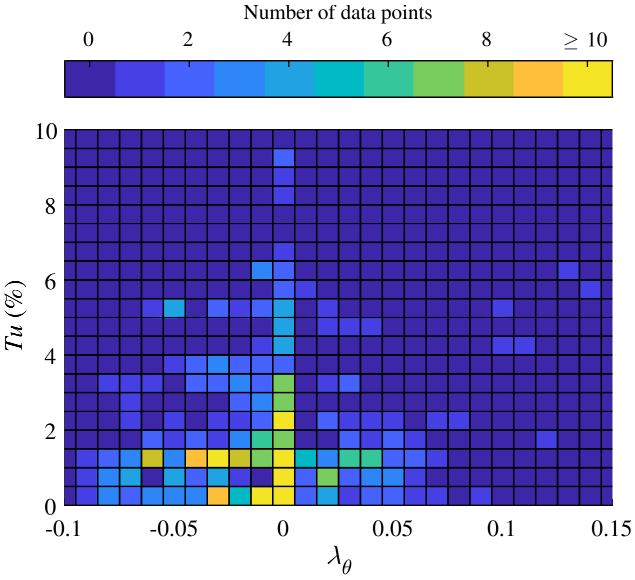

Fig. 5 **Histogram showing data points for Tu and λ_θ**

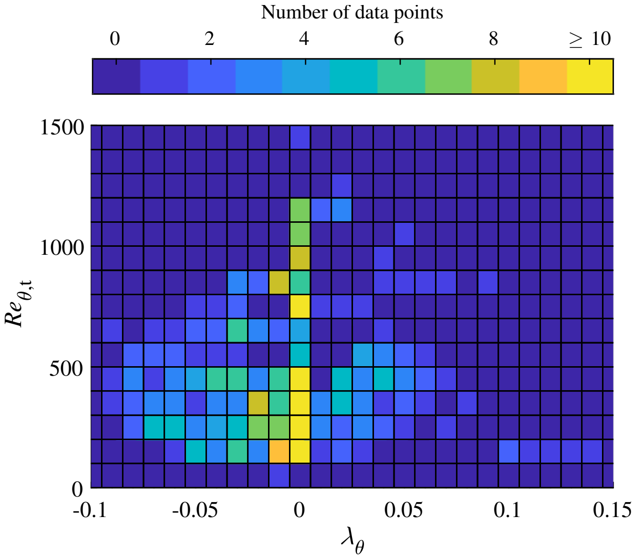

Fig. 6 **Histogram showing data points for $Re_{\theta,t}$ and λ_θ**

Figure 4 shows that there are large areas where there is no data to compare to correlations. This, along with the inherent scatter in the data, makes it difficult to adequately validate a model as predicted trends are hard to resolve in the data. Areas where the number of data points is lacking are shown by the histograms shown in Figs. 5 and 6. The $\lambda_\theta - Tu$ histogram in Fig. 5 shows where combinations of the two independent variables have not been investigated, while the $\lambda_\theta - Re_{\theta,t}$ histogram in Fig. 6 shows how the independent variables map onto the measured parameter. It can be seen that there is considerably less data available for favourable pressure gradients, especially those with strong accelerations where $\lambda_\theta > 0.1$. There is also little data for all pressure gradients with very high turbulence of $Tu < 5\,\%$. Figure 7 shows how, relatively, far each data point is from the value of $Re_{\theta,t}$ predicted by the Langtry and Menter correlation [43]. Data for $Tu < 3\,\%$ matches the correlations within 30 %, while for higher values of Tu errors grow to 40 % or more. Comparing Figs. 5, 6 and 7 to Fig. 1, it is clear that many flows relevant in turbomachinery have little experimental data backing up the correlations used in design.

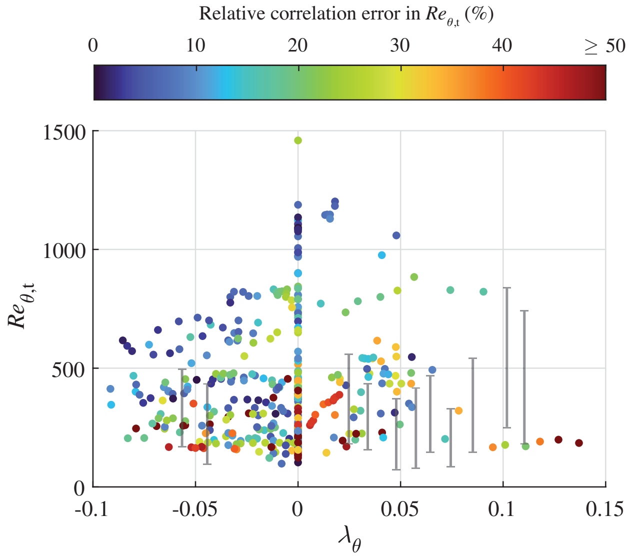

Fig. 7 **Relative error in $Re_{\theta,t}$ calculated by the correlation of Langtry and Menter [43] for each data point. Relevant turbomachinery cases from Fig. 1 are overlaid.**

5 Conclusions

This paper has consolidated measurements of transition onset from almost 70 years of research and presented them as a single database for the first time. The following conclusions, and suggestions, are drawn:

- Reporting of turbulence length scale and the location of measurements of turbulence intensity is inconsistent in the literature, particularly in older studies. Future investigations should seek to be consistent and complete in their reporting of these parameters.
- Data from all sources generally agrees well, with only a small number of significant outliers in the $Tu - \lambda_\theta - Re_{\theta,t}$ space.
- Correlations for $Re_{\theta,t}$ with and without pressure gradients capture overall trends well, however they do not fully describe the experimental data. For flows with pressure gradients, correlations extrapolate into regions for which there is limited or no data, making validation difficult.
- More data is needed across the $Tu - \lambda_\theta - Re_{\theta,t}$ space, but particularly in flows with high turbulence ($Tu < 5\,\%$) and favourable pressure gradients. Large systematic experimental campaigns that cover these regions while also collecting data for a wide range of Tu and λ_θ would help to limit the uncertainty that arises in comparing across data sets.
- As DNS becomes more accessible [47] it should be used in numerical experimental campaigns to provide additional data. DNS is able to provide a complete picture of the flow and may be used to probe mechanisms that are harder to resolve in physical experiments.
- Correlations and transition models used in CFD should continue to be updated and tuned to new data. A minimum requirement for any model should be to match the data shown in Fig. 4 as well as measurements on aerofoils.

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Data Availability Statement

The consolidated data set presented in this paper is available at whittle.digital/Transition_Database.

Nomenclature

K = acceleration parameter (-)
L = turbulence length scale (m)
Re = Reynolds number (-)
Tu = turbulence intensity (%)
U = boundary layer edge velocity ($\mathrm{m\,s^{-1}}$)
u' = fluctuating velocity ($\mathrm{m\,s^{-1}}$)
x = stream-wise coordinate (m)

Greek Letters

θ = momentum thickness $\int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)\mathrm{d}y$ (m)
λ = Thwaites' parameter (-)
ν = kinematic viscosity ($\mathrm{m^2\,s^{-1}}$)

Subscripts

t = transition onset value

Abbreviations

CFD = Computational Fluid Dynamics
DNS = Direct Numerical Simulation

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